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# Angular Correlations in Production Processes

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A simple and general method is suggested for studying processes in which resonances or unstable particles are produced. The method consists of analyzing the experimental data in terms of all the possible angular correlations among the decay products of the produced particles. The specific correlations that can be present in a number of experimentally feasible processes are explicitly tabulated. The usefulness of this method of analysis is illustrated by showing how the correlations provide extensive tests of various dynamical models of the production process such as one-particle exchanges. Independent of any models, a great deal can be learned about the production amplitude from just the correlations present when neither incident beam nor target are polarized, although, in general, additional correlations must be known to completely construct this amplitude.

## INTRODUCTION

THE increasing amount of data on processes involving the production and decay of unstable particles allows for the application of a simple and direct procedure for cataloging all possible information obtainable in the analysis of production reactions. The method consists of analyzing the processes in terms of all the angular correlations that can be present. A number of reactions are especially fruitful to study in this manner since the decay of the unstable particles involved can reveal considerable information concerning their spin, parity, and state of polarization. This information sheds light not only on the properties of the unstable particle itself but can be used to study the production process. This method of analyzing experiments, while being considerably simpler than performing a conventional phase-shift analysis, is a completely general description.

We have tabulated the kinds of angular correlations that can be present among the final particles in several production-decay processes which are experimentally feasible. In a number of instances, considerable data already exist.

Correlations of the type considered here have been studied for some time in the investigation of nuclear levels and the compound model of the nucleus.[1] In this paper we apply these ideas to the study of unstable particle production-decay processes.

Various dynamical models, such as one-particle-exchange approximations, can be tested by noting that these models imply that certain angular correlations either vanish or are related to others in a definite way. The implications of a number of current models of some of the reactions are included and tests of their validity are suggested. The usefulness of these model tests has already been demonstrated in a preliminary analysis of the process $K^-+p\to\Lambda+\omega$.[2]

Although the angular correlations which can be present when neither the incident beam nor the target are polarized are inadequate, in general, to allow one to construct the complete matrix element for the process, a surprising amount can be learned from such experiments. In some simple examples we show explicitly how to extract information about the production matrix element from the angular correlations when the initial particles are unpolarized. The additional correlations present in these examples when the target is polarized are also given in detail. A great deal more can be learned from these additional correlations, but presumably, experiments requiring a polarized target are more difficult to perform.

## ANGULAR CORRELATIONS

### A. $0^-+\frac{1}{2}^+\to\frac{1}{2}^++0^-$

Let us begin by considering processes of the type $0^-+\frac{1}{2}^+\to\frac{1}{2}^++0^-$, e.g., $\pi^-+p\to\Lambda+K^0$. (Spin and parity are denoted by $J^\pi$.) We will assume the target proton is unpolarized so that our present considerations will be independent of the initial state. The independent correlations present in this simple case are just the angular distribution of the final particles and the polarization of the $\Lambda$ normal to the production plane.

The most general matrix element for the process $\pi+p\to\Lambda+K$ can be expressed between free-particle spinors $\bar{u}(p_\Lambda)$ and $u(p_p)$ in the form[3]

$$\bar{u}(p_\Lambda)[a+b\gamma_5\gamma\cdot N]u(p_p),$$

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[1] See, for example, M. E. Rose and L. C. Biedenharn, Rev. Mod. Phys. 25, 729 (1953).

[2] S. M. Flatté, R. W. Huff, D. O. Huwe, F. T. Solmitz, and M. L. Stevenson, Bull. Am. Phys. Soc. 8, 603 (1963).

[3] We use natural units with $\hbar=c=1$. The metric is chosen so that the four vector product $a\cdot b=\mathbf{a}\cdot\mathbf{b}-a_0b_0$. Dirac spinors satisfy $(i\gamma\cdot p+m)u(p)=0$ and $\bar{u}(p)(i\gamma\cdot p+m)=0$, where $\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}$, $\gamma_\mu=\gamma_\mu^\dagger$, and $\gamma\cdot p=\boldsymbol{\gamma}\cdot\mathbf{p}+i\gamma_4E$. In addition, $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. Also, the abbreviation $\epsilon(abcd)=\epsilon_{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho d_\sigma$ will be used.

TABLE I. Angular correlations present in the process $K^-+p\rightarrow\Lambda+\omega$ are indicated by "$X$." Correlations excited by $K$ exchange, $K^*$ exchange, and their interference are indicated by ($K$), ($K^*$), and ($K+K^*$), respectively.

| | 1 | $(\hat{e}\cdot\hat{K}_\omega)^2$ | $(\hat{e}\cdot\hat{N})^2$ | $(\hat{e}\cdot\hat{K}_\omega)(\hat{e}\cdot\hat{L}_\omega)$ | $(\hat{e}\cdot\hat{K}_\omega)(\hat{e}\cdot\hat{N})$ | $(\hat{e}\cdot\hat{L}_\omega)(\hat{e}\cdot\hat{N})$ |
|---|---|---|---|---|---|---|
| 1 | $X$ ($K^*$) | $X$ ($K$),($K^*$) | $X$ ($K^*$) | $X$ | | |
| $\mathbf{W}\cdot\hat{N}$ | $X$ | $X$ | $X$ | $X$ ($K+K^*$) | | |
| $\mathbf{W}\cdot\hat{K}_\Lambda$ | | | | | $X$ ($K+K^*$) | $X$ |
| $\mathbf{W}\cdot\hat{L}_\Lambda$ | | | | | $X$ ($K+K^*$) | $X$ |

where $p_p$, $p_\Lambda$ are the momenta of the proton and lambda, respectively, and $N$ is a four-vector which reduces to a unit three-vector $\hat{N}$ normal to the production plane in the laboratory system. The amplitudes $a$ and $b$ are invariant functions of the incident energy and lambda production angle. The lambda polarization is then proportional to $\mathrm{Im}(ab^*)$ and would vanish for a simple model of production such as $K^*$ exchange. This is because the two amplitudes $a$ and $b$ are relatively real for such a model even though two independent amplitudes are necessary to describe the $K^*\Lambda p$ vertex. The reason for this is that analyticity does not allow either the electric or magnetic form factor to become complex in the scattering region (even in the presence of an anomalous threshold). The fact that the lambda cannot be polarized is then a straightforward prediction of a model in which the reaction is dominated by $K^*$ exchange. Analogous statements apply to other processes of this type such as $\pi+N\rightarrow N+\pi$, $\bar{K}+N\rightarrow\Sigma+\pi$, etc.

### B. $0^-+\frac{1}{2}^+\rightarrow\frac{1}{2}^++1^-$

A more interesting class of reactions are those of the type $0^-+\frac{1}{2}^+\rightarrow\frac{1}{2}^++1^-$, e.g., $K^-+p\rightarrow\Lambda+\omega$.[4] The cross section to produce $\Lambda+\omega$ is bilinear in the $\omega$ polarization vector $e$ and at most linear in the $\Lambda$ spin polarization $W$. (We mean by the $\omega$ polarization vector $e$ not the direction of its spin but rather the directions of the field components as in the usual description of a photon.) Since the $\omega$ has spin-parity $1^-$ its decay into $3\pi$'s is described by a matrix element of the form $e\cdot n$, where $e$ is the polarization vector of the $\omega$ and $n$ is a four-vector which reduces to the normal to the plane containing the three decay pions in their center of mass. Consequently, the subsequent $3\pi$ decay of the $\omega$ essentially measures $e$. Since $e$ is a vector and $W$ is a pseudovector, parity conservation in the production process dictates, in a simple manner, all possible correlations which can be analyzed by the decay channels of the $\Lambda$ and the $\omega$. In order to express these correlations in a convenient matrix form we introduce orthogonal coordinate systems in both the $\omega$ and $\Lambda$ rest frames. In the $\omega$ rest frame the basis vectors are chosen to be $\hat{K}_\omega$, a unit vector in the direction of the incident meson beam $\hat{N}$, a unit vector normal to the production plane, and $\hat{L}_\omega=\hat{K}_\omega\times\hat{N}$.

Similarly in the $\Lambda$ rest frame we have the orthogonal system $\hat{K}_\Lambda$, $\hat{N}$, $\hat{L}_\Lambda$. (Note that $\hat{K}_\Lambda$ and $\hat{K}_\omega$ as well as $\hat{L}_\omega$ and $\hat{L}_\Lambda$ are not the same vectors but are related through the Lorentz transformation connecting the $\Lambda$ and $\omega$ rest frames.)

The twelve possible correlations, consistent with parity conservation and Lorentz invariance, are conveniently denoted in Table I by an "$X$" in the appropriate box. The strengths of the various correlations are functions of the production angle and c.m. energy so that each entry corresponds to given set of production conditions.

One possible use of such a correlation table is to make a comparison with predictions of various dynamical models. For example, one can test the hypothesis that the process $K^-+p\rightarrow\Lambda+\omega$ is dominated by a linear combination of $K$ and $K^*$ exchanges.[5] Not all of the twelve correlations will be excited if the process is dominated by such poles. In fact only those correlations labeled by ($K$), ($K^*$), or ($K+K^*$) in Table I will be different from zero. The notation ($K$) indicates a contribution from $K$ exchange alone, ($K^*$) from $K^*$ exchange alone, and ($K+K^*$) indicates contributions from the interference between $K$ and $K^*$ exchanges. Moreover, there is one relation among the three ($K+K^*$) terms dictated by the model and hence only two of the three ($K+K^*$) terms are independent. One of the two independent correlations is

$$(\mathbf{W}\cdot\hat{p}_p)(\hat{e}\cdot\hat{K}_\omega)(\hat{e}\cdot\hat{N}),$$

where $\hat{p}_p$ is along the incident proton momentum in the

[4] Considerations, similar to those given here in this section, have been made by C. N. Yang and N. Byers (to be published).

[5] See also R. Huff, Phys. Rev. 133, B1078 (1964).

TABLE II. Angular correlations that can be present in the process $0^- + \frac{1}{2}^+ \to \frac{3}{2}^\pm(\to \frac{1}{2}^+ + 0^-) + 0^-$, e.g., $\pi^- + p \to Y^{*-}(\to \Lambda + \pi^-) + K^+$, are indicated by "±." The unit vector $\hat{p}_f$ is along the momentum of the decay spin-$\frac{1}{2}$ particle in the rest frame of the spin-$\frac{3}{2}$ particle. The polarization vector is $\mathbf{W}$. The vectors $\hat{K}$, $\hat{L}$, and $\hat{N}$ form a right-handed, orthonormal basis in the rest frame of the spin-$\frac{3}{2}$ particle, $\hat{K}$ being along the direction of the incident meson beam and $\hat{N}$ being normal to the production plane.

| | $1$ or $(\hat{p}_f\cdot\hat{K})^2$ or $(\hat{p}_f\cdot\hat{L})^2$ or $(\hat{p}_f\cdot\hat{K})(\hat{p}_f\cdot\hat{L})$ | $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{K})$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{L})$ | $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{K})^3$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{L})^3$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{K})(\hat{p}_f\cdot\hat{L})^2$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{L})(\hat{p}_f\cdot\hat{K})^2$ |
|---|---|---|---|
| $1$ or $(\mathbf{W}\cdot\hat{N})$ | ± | | |
| $(\mathbf{W}\cdot\hat{K})$ or $(\mathbf{W}\cdot\hat{L})$ | | ± | − |

$\Lambda$ rest frame. The other independent correlation is

$$(\hat{e}\cdot\hat{K}_\omega)(\hat{e}\times\hat{K}_\omega)\cdot\left[\mathbf{W}+\left(\frac{E_\Lambda-M_\Lambda}{M_\Lambda}\right)(\mathbf{W}\cdot\hat{p}_\Lambda)\hat{p}_\Lambda\right],$$

where $E_\Lambda$ and $\hat{p}_\Lambda$ are the $\Lambda$ energy and unit vector along the $\Lambda$ momentum in the $\omega$ rest frame, respectively, and $\mathbf{W}$ is the $\Lambda$ polarization. The combination in the bracket is $\mathbf{W}$ expressed in the $\omega$ rest frame. In addition, the contributions to the correlations 1 and $(\hat{e}\cdot\hat{K}_\omega)^2$ coming from just $K^*$ exchange are of equal magnitude but opposite in sign.

Note that if the process were dominated by a pure spin-zero exchange then eleven of the twelve correlations would have to be zero which is a more exhaustive test than the familiar Treiman-Yang test.[6]

### C. $0^- + \frac{1}{2}^+ \to 3/2^\pm(\to \frac{1}{2}^+ + 0^-) + 0^-$

Two examples of this reaction are the processes $\pi + p \to N^*(\to p + \pi) + \pi$ and $\pi + p \to Y^*(\to \Lambda + \pi) + K$.[7] We consider here only parity conserving decays of the spin-$\frac{3}{2}$ particle. If only the angular distribution of the $p+\pi(\Lambda+\pi)$ is measured then there are at most four possible correlations since no higher powers than $\cos^2\theta$ are permitted in a spin-$\frac{3}{2}$ decay in the $N^*$ or $Y^*$ rest system.[8] However if the decay proton or $\Lambda$ polarization is measured along with the decay distribution then $\cos^4\theta$ terms are permissible in the case of $\frac{3}{2}^-$, but only $\cos^2\theta$ terms in the case of $\frac{3}{2}^+$. The differences in correlations due to the different parities are easily seen from the general correlation table which can be constructed from the matrix element for the process.

This matrix element, in the rest system of the spin-$\frac{3}{2}$ particle, can be expressed between spinors of the initial and final spin-$\frac{1}{2}$ particles in the form

$$\bar{u}(p_f)\gamma_5(1\mp\beta)\{[\mathbf{p}_f\cdot\mathbf{K}-\tfrac{1}{3}(\boldsymbol{\gamma}\cdot\mathbf{p}_f)(\boldsymbol{\gamma}\cdot\mathbf{K})]\times[A_1^{(\pm)}+iA_2^{(\pm)}\gamma\cdot K]+[\mathbf{p}_f\cdot\mathbf{L}-\tfrac{1}{3}(\boldsymbol{\gamma}\cdot\mathbf{p}_f)(\boldsymbol{\gamma}\cdot\mathbf{L})]\times[A_3^{(\pm)}+iA_4^{(\pm)}\gamma\cdot K]\}u(p_i),$$

where $p_i$ and $p_f$ are the momenta of the initial and final spin-$\frac{1}{2}$ particles, $K$ is the initial meson momentum, and $\mathbf{L}$ is a linear combination of $\mathbf{p}_i$ and $\mathbf{K}$ which is orthogonal to $\mathbf{K}$ in rest system of the spin-$\frac{3}{2}$ particle. The upper (lower) signs refer to $\frac{3}{2}^+$ ($\frac{3}{2}^-$). The coefficients $A_1^{(\pm)}$–$A_4^{(\pm)}$ are complex invariant functions depending only on the production process, i.e., they depend on the initial energy and the angle at which the spin-$\frac{3}{2}$ particle is produced but not on the decay angle of the final spin-$\frac{1}{2}$ particle. The general angular distribution of the decay can be determined by squaring the above matrix element and averaging over initial target spins in which case the correlation matrix will take the form shown in Table II. Correlations possible for the $\frac{3}{2}^+$ case are labeled by + and for the $\frac{3}{2}^-$ case by −. The rows of this correlation matrix refer to the polarization directions of the decay spin-$\frac{1}{2}$ particle in the rest frame of the spin-$\frac{3}{2}$ resonance while the columns refer to the angular distribution of the decay spin-$\frac{1}{2}$ particle in the rest frame of the spin-$\frac{3}{2}$ resonance.

In the case of reactions such as $\pi + p \to Y^* + K$ and $K + p \to N^* + K$, etc., single pion or kaon exchange is not possible because of parity conservation. However, single vector-meson exchange is a possible model of such reactions. The $K^*$-exchange model predicts that the lambda in the reaction $\pi + p \to Y^*(\to \Lambda + \pi) + K$ would not be polarized for the same reasons as discussed in Sec. A. Similarly, the proton in the reaction $K + p \to N^*(\to p + \pi) + \pi$ could not be polarized if the exchange of a single $\rho$ dominated the process. The only possible correlations would then be the four terms in Table II for unpolarized decay spin-$\frac{1}{2}$ particle (i.e.,

[6] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

[7] Reactions of this type in connection with spin and parity determination have been considered by N. Byers and S. Fenster, Phys. Rev. Letters **11**, 52 (1963); R. H. Capps, Phys. Rev. **122**, 929 (1961); R. Gatto and H. P. Stapp, *ibid.* **121**, 1553 (1961); C. Itzykson and M. Jacob, Phys. Letters **3**, 153 (1963).

[8] C. N. Yang, Phys. Rev. **74**, 764 (1948).

TABLE III. Angular correlations that can be present in the process $0^- + \frac{1}{2}^+ \to \frac{3}{2}^\pm(\to \frac{1}{2}^+ + 0^-) + 1^-$, e.g., $\pi^- + p \to N^{*0}(\to p + \pi^-) + \omega$, are indicated by using "±." The unit vector $\hat{p}_f$ is along the momentum of the spin-$\frac{1}{2}$ particle in the rest frame of the spin-$\frac{3}{2}$ particle. The polarization vectors of the spin-$\frac{1}{2}$ and spin-1 particles are **W** and **e**, respectively. The vectors $\hat{K}$, $\hat{L}$, and $\hat{N}$ ($\hat{K}_\omega$, $\hat{L}_\omega$, and $\hat{N}_\omega$) form a right-handed orthonormal basis in the rest frame of the spin-$\frac{3}{2}$(1) particle, $\hat{K}(\hat{K}_\omega)$ being along the direction of the incident meson beam and $\hat{N}(\hat{N}_\omega)$ being normal to the production plane.

| | | $1$ or $(\hat{p}_f\cdot\hat{K})^2$ or $(\hat{p}_f\cdot\hat{L})^2$ or $(\hat{p}_f\cdot\hat{K})(\hat{p}_f\cdot\hat{L})$ | $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{K})$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{L})$ | $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{K})^3$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{L})^3$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{K})(\hat{p}_f\cdot\hat{L})^2$ or $(\hat{p}_f\cdot\hat{N})(\hat{p}_f\cdot\hat{L})(\hat{p}_f\cdot\hat{K})^2$ |
|---|---|---|---|---|
| $1$ or $\mathbf{W}\cdot\hat{N}$ | $1$ or $(\hat{e}\cdot\hat{K}_\omega)^2$ or $(\hat{e}\cdot\hat{L}_\omega)^2$ or $(\hat{e}\cdot\hat{K}_\omega)(\hat{e}\cdot\hat{L}_\omega)$ | ± | | |
| | $(\hat{e}\cdot\hat{N}_\omega)(\hat{e}\cdot\hat{K}_\omega)$ or $(\hat{e}\cdot\hat{N}_\omega)(\hat{e}\cdot\hat{L}_\omega)$ | | ± | − |
| $\mathbf{W}\cdot\hat{K}$ or $\mathbf{W}\cdot\hat{L}$ | $1$ or $(\hat{e}\cdot\hat{K}_\omega)^2$ or $(\hat{e}\cdot\hat{L}_\omega)^2$ or $(\hat{e}\cdot\hat{K}_\omega)(\hat{e}\cdot\hat{L}_\omega)$ | | ± | − |
| | $(\hat{e}\cdot\hat{N}_\omega)(\hat{e}\cdot\hat{K}_\omega)$ or $(\hat{e}\cdot\hat{N}_\omega)(\hat{e}\cdot\hat{L}_\omega)$ | ± | | |

the lambda or proton). All four of these correlations can be excited in the case of the most general interaction at the vertex of the spin-$\frac{1}{2}$, spin-1, and spin-$\frac{3}{2}$ particles.

Sakurai and Stodolsky[9] have suggested a special form of this latter interaction based on the "$\rho$-photon analogy" which leads to a reduction in the possible number of correlations. They show that for a "magnetic dipole $\rho$" the decay distribution in the $N^*$ rest system for the reaction $K+p \to N^* + K$ would be of the form

$$1+3(\hat{p}_f\cdot\hat{N})^2,$$

where $\hat{p}_f$ and $\hat{N}$ are unit vectors along the decay proton and normal to the production plane, respectively, in the $N^*$ rest system. Thus their model predicts only one independent correlation to be present out of a possible four. The required vanishing of the remaining three correlations provides a test of the model.

### D. $0^- + \frac{1}{2}^+ + \to 3/2^\pm(\to \frac{1}{2}^+ + 0^-) + 1^-$

Some examples of processes of this type are $K+p \to N^* + K^*$, $\bar{K}+p \to Y^* + \omega$, etc. We consider only parity conserving decays of the spin-$\frac{3}{2}$ particle. The analysis of the angular correlation combines the ideas of Secs. B and C, allowing the correlation table to be written down directly from Tables I and II. The conditions of parity conservation, linearity in the decay spin-$\frac{1}{2}$ polarization $W$, and bilinearity in the polarization vector $e$ of the produced spin-1 particle lead to the correlations summarized in Table III. In this table angular correlations possible for spin $\frac{3}{2}^\pm$ are indicated by "±." The polarization components $\mathbf{W}\cdot\hat{K}$, $\mathbf{W}\cdot\hat{L}$, and $\mathbf{W}\cdot\hat{N}$ of the spin-$\frac{1}{2}$ particle and the direction cosines of its momentum $\hat{p}_f\cdot\hat{K}$, $\hat{p}_f\cdot\hat{L}$, and $\hat{p}_f\cdot\hat{N}$ refer to the rest frame of the spin-$\frac{3}{2}$ particle. Similarly, the components of the polarization vector $\hat{e}\cdot\hat{K}_\omega$, $\hat{e}\cdot\hat{L}_\omega$, and $\hat{e}\cdot\hat{N}_\omega$ of the spin-1 particle refer to its rest system. The vector $\hat{e}$ is the normal to the plane of the $3\pi$'s in $\omega$ decay or is along one of the decay mesons in either $\rho$ decay or $K^*$ decay.

In reactions like $K+p \to N^* + K^*$ and $\pi + p \to N^* + \rho$ where single-pion exchange could be the dominating mechanism the correlation table will contain only a few terms. In fact for such a mechanism there can be no polarization of the decay spin-$\frac{1}{2}$ particle and the decay angular distribution will be of the form

$$[1+3(\hat{q}\cdot\hat{p})^2](\hat{e}\cdot\hat{K}_\omega)^2,$$

where $\hat{q}$ and $\hat{p}$ are unit vectors along the directions of the initial and final spin-$\frac{1}{2}$ particles, respectively, in the rest system of the spin-$\frac{3}{2}$ particle. Hence the pure single-pion-exchange model predicts 116 (68) of the possible 120 (72) correlations to be zero for spin $\frac{3}{2}^+$ ($\frac{3}{2}^-$).

We remark that correlation tables for producing a spin-2 particle, decaying into 2 spin-zero bodies, rather than spin one can be easily written down in analogy to Tables II and III. The only difference here is that the

[9] J. J. Sakurai and L. Stodolsky, Phys. Rev. Letters **11**, 90 (1963).

TABLE IV. Possible angular correlations present in the process $p\bar{p}\to\Lambda\bar{\Lambda}$ are indicated by "X". Those correlations excited by $K$ exchange, $K^*$ exchange, and their interference are indicated by $(K)$, $(K^*)$, and $(K+K^*)$, respectively.

| | 1 | $(\mathbf{W}_\Lambda\cdot\hat{N}_{\bar{\Lambda}})$ | $(\mathbf{W}_\Lambda\cdot\hat{K}_{\bar{\Lambda}})$ | $(\mathbf{W}_\Lambda\cdot\hat{L}_{\bar{\Lambda}})$ |
|---|---|---|---|---|
| 1 | $X$ $(K),(K^*)$ | $X$ | | |
| $(\mathbf{W}_\Lambda\cdot\hat{N}_\Lambda)$ | $X$ | $X$ $(K^*),(K+K^*)$ | | |
| $(\mathbf{W}_\Lambda\cdot\hat{K}_\Lambda)$ | | | $X$ $(K^*),(K+K^*)$ | $X$ $(K^*),(K+K^*)$ |
| $(\mathbf{W}_\Lambda\cdot\hat{L}_\Lambda)$ | | | $X$ $(K^*),(K+K^*)$ | $X$ $(K^*),(K+K^*)$ |

entries referring to the spin-2 particle should be quadrilinear in the decay momentum rather than bilinear as in the spin-1 case.

### E. $\frac{1}{2}^+ + \frac{1}{2}^+ \to \frac{1}{2}^+(\to \frac{1}{2}^+ + 0^-) + \frac{1}{2}^+(\to \frac{1}{2}^+ + 0^-)$

Here[10] we have in mind processes like $p+\bar{p}\to\Lambda+\bar{\Lambda}$, $\Sigma+\bar{\Sigma}$, etc. The correlation table for this case, Table IV, can be easily written down in a similar manner to Table II. The condition of parity conservation in the production process leads to the eight possible correlations which are marked in Table IV. By charge conjugation symmetry only six of the possible eight are independent.

Possible mechanisms for processes like $p+\bar{p}\to\Lambda+\bar{\Lambda}$ might be a combination of $K$ and $K^*$ exchanges similar to the model in the case of the reaction $K^-+p\to\Lambda+\omega$. In this case six of the possible eight correlations are excited and are indicated with the same notation as in Table II. One sees that there is a simple test for such a model involving only the components of the $\Lambda$ polarization perpendicular to the production plane which should be zero if the process is dominated by $K$ and $K^*$ exchanges.

### F. $\frac{1}{2}^+ + \frac{1}{2}^+ \to 3/2^\pm(\to \frac{1}{2}^+ + 0^-) + 3/2^\pm(\to \frac{1}{2}^+ + 0^-)$

Examples of such reactions are $p+\bar{p}\to N^*+\bar{N}^*$ or $p+\bar{p}\to Y^*+\bar{Y}^*$. As before, parity conservation in the production and decay as well as at most quadrilinearity in the decay spin-$\frac{1}{2}$ momentum and linearity in the decay spin-$\frac{1}{2}$ polarization dictates the allowable correlations (indicated by $X$ in Table V). Because of the possibility of measuring the polarization of the decay spin-$\frac{1}{2}$ particle this table is very large and is essentially the product of Table II with itself. One has then 800 allowable correlations out of a possible 1600.

A process such as $p+\bar{p}\to N^*+\bar{N}^*$ dominated by single-pion exchange would lead to only unpolarized final spin-$\frac{1}{2}$ particles and an angular distribution of decay products of $\bar{N}^*N^*$ of the form

$$[1+3(\hat{q}'\cdot\hat{K}')^2]\times[1+3(\hat{q}\cdot\hat{K})^2],$$

where $\hat{q}'$ is a unit vector along the decay antiproton momentum, $\hat{K}'$ is a unit vector along the incident $\bar{p}$ both in the $\bar{N}^*$ rest frame; $\hat{q}$ is a unit vector along the decay proton and $\hat{K}$ is a unit vector along the incident proton in the $N^*$ rest system. Thus, the one-pion-exchange (OPE) model would show only 4 out of a possible 800 correlations.

## PRODUCTION AMPLITUDES

### A. $0^- + \frac{1}{2}^+ \to \frac{1}{2}^+ + 0^-$

Let us consider[11] the process $\pi^-+p\to\Lambda+K^0$ for definiteness. The matrix element for a process of this type, can be written in the form $\bar{u}(p_\Lambda)[a+b\gamma_5\gamma\cdot N]\times u(p_p)$, where $a$ and $b$ are complex invariant functions of energy and angle, $N$ is a four vector which reduces to a unit three-vector $\hat{N}$ normal to the production plane in the laboratory system, and $p_p$ and $p_\Lambda$ are the proton and lambda four-momenta, respectively. Since the over-all phase is arbitrary, only the relative phase of $a$ and $b$ is of importance. The transition probability between proton and lambda polarization states described by the four-vector $W_p$ and $W_\Lambda$ is obtained simply by squaring the matrix element. This result is[12]

$$|A|^2=(|a|^2+|b|^2)+2\,\mathrm{Im}(a^*b)(\mathbf{W}_\Lambda\cdot\hat{N}+\mathbf{W}_p\cdot\hat{N})$$
$$+2|b|^2(\mathbf{W}_p\cdot\hat{N})(\mathbf{W}_\Lambda\cdot\hat{N})+(|a|^2-|b|^2)(\mathbf{W}_p\cdot\mathbf{W}_\Lambda)$$
$$+2\,\mathrm{Re}(a^*b)M_pM_\Lambda(M_pM_\Lambda-p_\Lambda p_p)^{-1}(\hat{N}\cdot\mathbf{W}_p\times\mathbf{W}_\Lambda),$$

where $\mathbf{W}_\Lambda$ and $\mathbf{W}_p$ are the lambda and proton polarizations in their respective rest frames.

[10] Angular correlations in the process $\bar{p}+p\to\bar{\Lambda}+\Lambda$ where neither beam nor target are polarized have been given explicitly by C. H. Chan, Phys. Rev. **133**, B431 (1964).

[11] Considerations similar to this section have been employed in the nucleon-nucleon problem by R. Schumacher and H. Bethe, Phys. Rev. **121**, 1534 (1961).

[12] The normalization of the amplitude $A$ can be chosen such that the differential cross section in the center of mass is simply given by $d\sigma/d\Omega=|A|^2$.

It is clear from the above equation that there are a total of four independent correlations which we can take to be:

(i) the angular distribution, which gives the term $|a|^2+|b|^2$;

(ii) the lambda polarization perpendicular to the production plane, which gives $\mathrm{Im}(a^*b)$;

(iii) the lambda polarization along the initial proton polarization which gives $|a|^2-|b|^2$;

(iv) the lambda polarization along the normal to the plane containing the incident proton polarization and the normal to the production plane, which gives $\mathrm{Re}(a^*b)$.

It is clear that $|a|$, $|b|$, and the relative phase of $a$ and $b$ can be obtained easily from these and that all four correlations are necessary for a unique solution.

Without a polarized target only correlations (i) and (ii) are present and the maximum information obtainable is $|a|^2+|b|^2$ and $\mathrm{Im}(a^*b)$.

### B. $0^-+\frac{1}{2}^+ \to \frac{1}{2}^+ + 1^-$

Let us consider the process $K^-+p\to\Lambda+\omega$ for definiteness. The most general form of the matrix element for a process of this type can be put in the form

$$\begin{aligned}\bar{u}(p')\{&[a_1 e\cdot K\gamma_5+a_3 e\cdot L\gamma_5+a_5 e\cdot N](1+i\gamma_5\gamma\cdot N)\\&+[a_2 e\cdot K\gamma_5+a_4 e\cdot L\gamma_5+a_6 e\cdot N](1-i\gamma_5\gamma\cdot N)\}u(p)\,,\end{aligned}$$

where $a_1$–$a_6$ are invariant functions of energy and angle and $p$, $p'$, $K$, and $q$ are the four-momenta of the proton, lambda, kaon, and omega, respectively. In the $\omega$ rest frame the space-like components of $K$, $L$, and $N$ reduce to an orthogonal coordinate system such that $\hat{N}$ is a unit vector normal to the production plane, $\hat{K}$ is a unit vector along the beam direction, and $\hat{L}=\hat{N}\times\hat{K}$. The vector $e$ describes the $\omega$ field and satisfies $q\cdot e=0$ and $e^2=1$.

Taking the matrix element between proton and lambda polarization states described by four vectors $W$ and $W'$, respectively, and squaring gives the following transition rate[12]:

$$\begin{aligned}
|A|^2=&[(|a_1|^2+|a_2|^2)(e\cdot K)^2+(|a_3|^2+|a_4|^2)(e\cdot L)^2\\
&+2\,\mathrm{Re}(a_1a_3^*+a_2a_4^*)(e\cdot K)(e\cdot L)](-m_pm_\Lambda-p\cdot p')+(|a_5|^2+|a_6|^2)(e\cdot N)^2(m_pm_\Lambda-p\cdot p')\\
&+2\,\mathrm{Re}(a_3a_6^*+a_4a_5^*)m_p(W\cdot p')(e\cdot L)(e\cdot N)-2\,\mathrm{Re}(a_1a_5^*+a_2a_6^*)m_\Lambda(W'\cdot p)(e\cdot K)(e\cdot N)\\
&+2\,\mathrm{Re}(a_1a_6^*+a_2a_5^*)m_p(W\cdot p')(e\cdot K)(e\cdot N)-2\,\mathrm{Re}(a_3a_5^*+a_4a_6^*)m_\Lambda(W'\cdot p)(e\cdot L)(e\cdot N)\\
&+(|a_2|^2-|a_1|^2)(e\cdot K)^2(m_\Lambda m_p+p\cdot p')[(W-W')\cdot N]+(|a_4|^2-|a_3|^2)(e\cdot L)^2(m_\Lambda m_p+p\cdot p')[(W-W')\cdot N]\\
&+2\,\mathrm{Re}(a_2a_4^*-a_1a_3^*)(e\cdot K)(e\cdot L)(m_\Lambda m_p+p\cdot p')[(W-W')\cdot N]\\
&+(|a_6|^2-|a_5|^2)(e\cdot N)^2(m_\Lambda m_p+p\cdot p')[(W+W')\cdot N]+2i\,\mathrm{Im}(a_2a_6^*-a_1a_5^*)(e\cdot K)(e\cdot N)\epsilon(pp'NW')\\
&+2i\,\mathrm{Im}(a_4a_6^*-a_3a_5^*)(e\cdot L)(e\cdot N)\epsilon(pp'NW')+2i\,\mathrm{Im}(a_2a_5^*-a_1a_6^*)(e\cdot K)(e\cdot N)\epsilon(pp'NW)\\
&+2i\,\mathrm{Im}(a_4a_5^*-a_3a_6^*)(e\cdot L)(e\cdot N)\epsilon(pp'NW)+2\,\mathrm{Re}a_1a_2^*(e\cdot K)^2(W\cdot W')(m_\Lambda m_p+p\cdot p')\\
&+2\,\mathrm{Re}a_3a_4^*(e\cdot L)^2(W\cdot W')(m_\Lambda m_p+p\cdot p')+2\,\mathrm{Re}(a_1a_4^*+a_2a_3^*)(e\cdot K)(e\cdot L)(W\cdot W')(m_\Lambda m_p+p\cdot p')\\
&+2\,\mathrm{Re}a_5a_6^*(e\cdot N)^2(W\cdot W')(m_\Lambda m_p-p\cdot p')-2\,\mathrm{Re}(a_1a_2^*)(e\cdot K)^2(W\cdot p')(W'\cdot p)-2\,\mathrm{Re}a_3a_4^*(e\cdot L)^2(W\cdot p')(W'\cdot p)\\
&-2\,\mathrm{Re}(a_1a_4^*+a_2a_3^*)(e\cdot K)(e\cdot L)(W\cdot p')(W'\cdot p)+2\,\mathrm{Re}(a_5a_6^*)(e\cdot N)^2(W\cdot p')(W'\cdot p)\\
&+|(a_1-a_2)(e\cdot K)+(a_3-a_4)(e\cdot L)|^2(W\cdot N)(W'\cdot N)(m_pm_\Lambda+p\cdot p')+|a_5-a_6|^2(e\cdot N)^2(W\cdot N)(W'\cdot N)(m_pm_\Lambda-pp')\\
&+2\,\mathrm{Re}(a_2a_6^*-a_1a_5^*)(e\cdot K)(e\cdot N)(W\cdot N)(W'\cdot p)m_\Lambda+2\,\mathrm{Re}(a_4a_6^*-a_3a_5^*)(e\cdot L)(e\cdot N)(W\cdot N)(W'\cdot p)m_\Lambda\\
&+2\,\mathrm{Re}(a_2a_5^*-a_1a_6^*)(e\cdot K)(e\cdot N)(W'\cdot N)(W\cdot p')m_p+2\,\mathrm{Re}(a_4a_5^*-a_3a_6^*)(e\cdot L)(e\cdot N)(W'\cdot N)(W\cdot p')m_p\\
&-2i\,\mathrm{Im}(a_1a_6^*+a_2a_5^*)(e\cdot K)(e\cdot N)\epsilon(pp'WW')-2i\,\mathrm{Im}(a_3a_6^*+a_4a_5^*)(e\cdot L)(e\cdot N)\epsilon(pp'WW')\\
&+2i\,\mathrm{Im}(a_5a_6^*)(e\cdot N)^2[m_\Lambda\epsilon(pNWW')+m_p\epsilon(p'NW'W)]-2\,i\mathrm{Im}(a_1a_2^*)(e\cdot K)^2[m_\Lambda\epsilon(pNWW')+m_p\epsilon(p'NW'W)]\\
&-2i\,\mathrm{Im}(a_3a_4^*)(e\cdot L)^2[m_\Lambda\epsilon(pNWW')+m_p\epsilon(p'NW'W)]\\
&+2i\,\mathrm{Im}(a_2a_3^*-a_1a_4^*)(e\cdot K)(e\cdot L)[m_\Lambda\epsilon(pNWW')+m_p\epsilon(p'NW'W)]\\
&+i\,\mathrm{Im}[(a_1-a_2)(a_5^*-a_6^*)](e\cdot K)(e\cdot N)[(W\cdot N)\epsilon(NW'p'p)+(W'\cdot N)\epsilon(NWpp')]\\
&\qquad+i\,\mathrm{Im}[(a_3-a_4)(a_5^*-a_6^*)](e\cdot L)(e\cdot N)[(W\cdot N)\epsilon(NW'p'p)+(W'\cdot N)\epsilon(NWpp')].
\end{aligned}$$

As can be seen from this expression there are a total of 48 correlations present when the target is polarized. However, only 32 are linearly independent.

The maximum information one can obtain with an unpolarized target, as can be seen by putting $W=0$ in the above equation is given by the following 12 correlations:

(i) $|a_1|^2+|a_2|^2$
(ii) $|a_1|^2-|a_2|^2$
(iii) $|a_3|^2+|a_4|^2$
(iv) $|a_3|^2-|a_4|^2$
(v) $|a_5|^2+|a_6|^2$
(vi) $|a_5|^2-|a_6|^2$
(vii) $\mathrm{Re}(a_1a_3^*+a_2a_4^*)$
(viii) $\mathrm{Re}(a_1a_3^*-a_2a_4^*)$
(ix) $\mathrm{Re}(a_1a_5^*+a_2a_6^*)$
(x) $\mathrm{Im}(a_1a_5^*-a_2a_6^*)$
(xi) $\mathrm{Re}(a_3a_5^*+a_4a_6^*)$
(xii) $\mathrm{Im}(a_3a_5^*-a_4a_6^*)$.

TABLE V. Angular correlations for the process $p+\bar{p}\rightarrow Y^*(\rightarrow\Lambda\pi)+\bar{Y}^*(\rightarrow\bar{\Lambda}\pi)$. The rows refer to the angular distribution and polarization distribution of the $\Lambda$ in the $Y^*$ rest frame and similarly the columns refer to the $\bar{\Lambda}$ in the $\bar{Y}^*$ rest frame. Allowed correlations are indicated by $X$. All vectors except $\mathbf{W}$ and $\mathbf{W}'$ are unit vectors.

| Particle correlations \ Antiparticle correlations | | $1$ or $(\mathbf{q}'\cdot\mathbf{K}')^2$ or $(\mathbf{q}'\cdot\mathbf{L}')^2$ or $(\mathbf{q}'\cdot\mathbf{K}')(\mathbf{q}'\cdot\mathbf{L}')$ | | | | $(\mathbf{q}'\cdot\mathbf{K}')(\mathbf{q}'\cdot\mathbf{N}')$ or $(\mathbf{q}'\cdot\mathbf{L}')(\mathbf{q}'\cdot\mathbf{N}')$ or $(\mathbf{q}'\cdot\mathbf{K}')^3(\mathbf{q}'\cdot\mathbf{N})$ or $(\mathbf{q}'\cdot\mathbf{L}')^3(\mathbf{q}'\cdot\mathbf{N}')$ or $(\mathbf{q}'\cdot\mathbf{K}')^2(\mathbf{q}'\cdot\mathbf{L}')(\mathbf{q}'\cdot\mathbf{N}')$ or $(\mathbf{q}'\cdot\mathbf{L}')^2(\mathbf{q}'\cdot\mathbf{K}')(\mathbf{q}'\cdot\mathbf{N}')$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | $\mathbf{W}'\cdot\mathbf{N}'$ | $\mathbf{W}'\cdot\mathbf{K}'$ | $\mathbf{W}'\cdot\mathbf{L}'$ | 1 | $\mathbf{W}'\cdot\mathbf{N}'$ | $\mathbf{W}'\cdot\mathbf{K}'$ | $\mathbf{W}'\cdot\mathbf{L}'$ |
| $1$ or $(\mathbf{q}\cdot\mathbf{K})^2$ or $(\mathbf{q}\cdot\mathbf{L})^2$ or $(\mathbf{q}\cdot\mathbf{K})(\mathbf{q}\cdot\mathbf{L})$ | 1 | $X$ | $X$ | | | | | $X$ | $X$ |
| | $\mathbf{W}\cdot\mathbf{N}$ | $X$ | $X$ | | | | | $X$ | $X$ |
| | $\mathbf{W}\cdot\mathbf{K}$ | | | $X$ | $X$ | $X$ | $X$ | | |
| | $\mathbf{W}\cdot\mathbf{L}$ | | | $X$ | $X$ | $X$ | $X$ | | |
| $(\mathbf{q}\cdot\mathbf{K})(\mathbf{q}\cdot\mathbf{N})$ or $(\mathbf{q}\cdot\mathbf{L})(\mathbf{q}\cdot\mathbf{N})$ or $(\mathbf{q}\cdot\mathbf{K})^3(\mathbf{q}\cdot\mathbf{N})$ or $(\mathbf{q}\cdot\mathbf{L})^3(\mathbf{q}\cdot\mathbf{N})$ or $(\mathbf{q}\cdot\mathbf{K})^2(\mathbf{q}\cdot\mathbf{L})(\mathbf{q}\cdot\mathbf{N})$ or $(\mathbf{q}\cdot\mathbf{L})^3(\mathbf{q}\cdot\mathbf{K})(\mathbf{q}\cdot\mathbf{N})$ | 1 | | | $X$ | $X$ | $X$ | $X$ | | |
| | $\mathbf{W}\cdot\mathbf{N}$ | | | $X$ | $X$ | $X$ | $X$ | | |
| | $\mathbf{W}\cdot\mathbf{K}$ | $X$ | $X$ | | | | | $X$ | $X$ |
| | $\mathbf{W}\cdot\mathbf{L}$ | $X$ | $X$ | | | | | $X$ | $X$ |

The first six correlations determine the magnitudes of each amplitude. Since the over-all phase is arbitrary, there remain five independent phases which unfortunately are not uniquely determined by the last six correlations. The degree of ambiguity will depend on the actual values of the correlations. For example, should it happen that only $a_1$ and $a_3$ or only $a_2$ and $a_4$ were substantially different from zero in some region of energy, the sign of their relative phase would be the only ambiguity occurring in the construction of the amplitude.

In general, the solutions for the phases can be obtained in the following manner: Let $a_n=|a_n|\exp(i\theta_n)$ and choose $\theta_6=0$ by adjusting the arbitrary over-all phase. From correlations (vii) and (viii) one can determine $\cos(\theta_1-\theta_3)$ and $\cos(\theta_2-\theta_4)$. For each choice of the signs of $\theta_1-\theta_3$ and $\theta_2-\theta_4$, one can solve the remaining relations for $\theta_2$, $\theta_4$, $\theta_3-\theta_5$, and $\theta_1-\theta_5$. In general, one can go no further.

We see the correlations that are present when the target is unpolarized do not determine the production amplitudes uniquely but, in general, lead to several sets of amplitudes, the extent of the indeterminancy depending on the actual values of the correlations. Although it is clearly necessary to measure some correlations that can be present only when the target is polarized to completely remove these ambiguities, more than one might anticipate can be learned even with an unpolarized target.

In certain cases discussed above involving the production and decay of spin-$\frac{3}{2}$ particles, the possible number of correlations far exceeds the number of independent amplitudes even for an unpolarized target. It is an interesting question whether it is possible, in these cases, to learn the entire matrix element without the use of a polarized target.

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